[This question paper contains 4 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 6623 **HC**

Unique Paper Code : 32351301

Name of the Paper : Theory of Real Functions

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1. (a) Use the ϵ-δ definition of the limit to find $\lim_{x \to 2} f(x)$ where $f(x) = \frac{1}{1-x}$. (5)

   (b) State and prove Sequential Criterion for Limits. (5)

   (c) State Squeeze Theorem. For $n \in \mathbb{N}$, $n \geq 3$, derive the inequality, $-x^2 \leq x^n \leq x^2$ for $-1 < x < 1$. Hence prove that $\lim_{x \to 0} x^n = 0$ for $n \geq 3$, assuming that $\lim_{x \to 0} x^2 = 0$. (5)

P.T.O.

(d) Let f, g be defined on $A \subseteq R$ to R, and let c be a cluster point of A. Suppose that f is bounded on a neighbourhood of c and that $\lim_{x \to c} g = 0$. Prove that $\lim_{x \to c} fg = 0$. (5)

2. (a) Let $c \in R$ and let f be defined for $x \in (c, \infty)$ and $f(x) > 0$ for all $x \in (c, \infty)$. Show that $\lim_{x \to c} f = \infty$ if and only if $\lim_{x \to c} \frac{1}{f} = 0$. (5)

(b) Prove that

(i) $\lim_{x \to 0} \frac{1}{\sqrt{|x|}} = +\infty, \ x \neq 0$

(ii) $\lim_{x \to 0-} e^{1/x} = 0, \ x \neq 0$. (5)

(c) Let A = R and let f be Dirichlet's function defined by

$$g(x) = \begin{cases} 1, & \text{for x rational} \\ -1, & \text{for x irrational} \end{cases}$$

Show that f is discontinuous at any point of R. (5)

(d) Let $f : R \to R$ be continuous at c and let $f(c) > 0$. Show that there exists a neighbourhood $V_\delta(c)$ of c such that if $x \in V_\delta(c)$ then $f(x) > 0$. (5)

3. (a) Determine the points of continuity of the function $f(x) = x - [\![x]\!]$, $x \in R$, where $[\![x]\!]$ denotes the greatest integer $n \in \mathbb{Z}$ such that $n \le x$. (5)

(b) Let $A, B \subseteq R$, let $f: A \to R$ be continuous on A, and let $g: B \to R$ be continuous on B. If $f(A) \subseteq B$, show that the composite function $gof: A \to R$ is continuous on A. (5)

(c) Let f be a continuous real valued function defined on [a, b]. Show that f is a bounded function. (5)

(d) Prove that a polynomial of odd degree has at least one real root. (5)

4. (a) Define uniform continuity of a function on a set $A \subseteq R$. Show that every uniformly continuous function on A is continuous on A. Is the converse true? Justify your answer. (5)

(b) Show that the function $\sqrt{x}$ is uniformly continuous on $[0, \infty)$. (5)

(c) Let I, J be intervals in R, let $g: I \to R$ and $f: J \to R$ be functions such that $f(J) \subseteq I$ and let $c \in J$. If f is differentiate at c and if g is differentiate at f(c), show that the composite function gof is differentiate at c and $(gof)'(c) = g'(f(c)).f'(c)$. (5)

(d) Let $f: R \rightarrow R$ be defined by

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that f is differentiable at $x = 0$ and find $f'(0)$. (5)

5. (a) Let f be continuous on [a, b] and differentiate on (a, b). Prove that f is increasing on [a, b] if and only if $f'(x) \geq 0 \ \forall x \in [a, b]$. (5)

(b) State Darboux's Theorem. Suppose that if $f : [0,2] \rightarrow R$ is continuous on [0, 2] and differentiate on (0, 2), and that $f(0) = 0$, $f(1) = 1$, $f(2) = 1$.

(i) Show that there exists $c_1 \in (0,1)$ such that $f'(c_1) = 1$

(ii) Show that there exists $c_2 \in (1,2)$ such that $f'(c_2) = 0$

(iii) Show that there exists $c \in (0,2)$ such that $f'(c) = 1/3$. (5)

(c) Find the Taylor series for cos x and indicate why it converges to cos x $\forall x \in R$. (5)

(d) Define a convex function on [a, b]. Check the convexity of the following functions on given intervals :

(i) $f(x) = x - \sin x$, $x \in [0, \pi]$.

(ii) $g(x) = x^3 + 2x$, $x \in [-1, 1]$. (5)

[This question paper contains 4 printed pages.]

(4) 13/12/17

Your Roll No..............

Sr. No. of Question Paper : **6624** **HC**

Unique Paper Code : 32351302

Name of the Paper : Group Theory 1

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory.

1. (a) For a fixed point (a, b) in $\mathbb{R}^2$, define $T_{(a,b)} : \mathbb{R}^2 \to \mathbb{R}^2$ by $(x, y) \to (x + a, y + b)$.

   Show that $T(\mathbb{R}^2) = \{T_{a,b} \mid a, b \in \mathbb{R}\}$

   is a group under function composition. (6)

   (b) (i) Find the inverse of $\begin{pmatrix} 2 & 6 \\ 3 & 5 \end{pmatrix}$ in $GL(2, \mathbb{Z}_{11})$. (4)

(ii) Let G be an Abelian group under multiplication with identity e. Show that

$H = \{x^2 \mid x \in G\}$ is a subgroup of G. (2)

(c) (i) Let G be a group. Show that $Z(G) = \bigcap_{a \in G} C(a)$

where Z(G) is the Center of G and C(a) is the Centralizer of a. (4)

(ii) Let G be the group of nonzero real numbers under multiplication. Show that

$$H = \{x \in G \mid x = 1 \text{ or } x \text{ is irrational}\}$$

and $K = \{x \in G \mid x \geq 1\}$ are not subgroups of G. (2)

2. (a) Let G be a group and let $a \in G$. If $|a| = n$, prove that $\langle a \rangle = \{e, a, a^2, \ldots, a^{n-1}\}$ and $a^i = a^j$ if and only if n divides $i - j$. (6)

(b) Suppose that $|a| = 24$. Find a generator for $\langle a^{21} \rangle \cap \langle a^{10} \rangle$. (6)

(c) If $|a| = n$, show that

$$\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle$$

and that $$|a^k| = \frac{n}{\gcd(n,k)}$$ (6)

3. (a) Define the Alternating Group A_n. Show that it forms a subgroup of the Permutation Group S_n and $|A_n| = \frac{n!}{2}$. (6)

(b) Prove that every group is isomorphic to a group of permutations. (6)

(c) Prove that U(10) is not isomorphic to U(12). (6)

4. (a) State and prove Orbit Stabilizer Theorem. (6½)

(b) (i) Prove that $aH = H$ if and only if $a \in H$. (3)

(ii) Prove that $aH = bH$ or $aH \cap bH = \phi$. (3½)

(c) (i) Prove that order of U(n) is even when $n > 2$. (3)

(ii) Prove that a group of prime order is cyclic. (3½)

5. (a) Let H and K be subgroups of a finite group G and let

$$HK = \{hk \mid h \in H,\ k \in K\}$$

and $$KH = \{kh \mid k \in K,\ h \in H\}.$$

Prove that HK is a group if and only if $HK = KH$. (6½)

(b) Let φ be a homomorphism from a group G to a group $\bar{G}$ and let g be an element of G. Prove that

(i) If $\varphi(g) = g'$, then $\varphi^{-1}(g') = \{x \in G | \varphi(x) = g'\} = g\text{Ker}\varphi$ (4)

(ii) If $|\text{Ker}\varphi| = n$, then φ is an $n - \text{to} - 1$ mapping from G onto $\varphi(G)$. (2½)

(c) (i) Prove that A_n is normal in S_n. (3½)

(ii) If G is a non-Abelian group of order p^3 (p is prime) and $Z(G) \neq \{e\}$, prove that $|Z(G)| = p$. (3)

6. (a) State and prove The First Isomorphism Theorem. (6½)

(b) Let G be a group and let Z(G) be the center of G. Prove that if $G/Z(G)$ is cyclic, then G is Abelian. (6½)

(c) Let $4Z = (0, \pm 4, \pm 8, \cdots\}$. Find $Z/4Z$. (6½)

(5) 16/12/17

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **6625**

Unique Paper Code : **32351303** **HC**

Name of the Paper : **Multivariate Calculus**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

Attempt any *five* questions from each Section.

All questions carry equal marks.

Section I

1. Let f be the function defined by :

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x^2 + y^6} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$$

Is f continuous at $(0, 0)$? Explain.

2. Find the equation for each horizontal tangent plane to the surface :

$$z = 5 - x^2 - y^2 + 4y.$$

3. Let f and g be twice differentiable functions of one variable and let $u(x, t) = f(x + ct) + g(x - ct)$ for a constant c. Show that :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

4. Let f have continuous partial derivatives and suppose the maximal directional derivative of f at $P_0(1, 2)$ has magnitude 50 and is attained in the direction from P_0 towards Q(3, –4). Use this information to find $\nabla f(1, 2)$.

5. Find the absolute extrema of $f(x, y) = x^2 + xy + y^2$ on the closed bounded set S where S is the disk $x^2 + y^2 \leq 1$.

6. Find the point on the plane $2x + y + z = 1$ that is nearest to the origin.

Section II

7. Find the area of the region D by setting double integral, where D is bounded by the parabola $y = x^2 - 2$ and the line $y = x$.

8. Write an equivalent integral with the order of integration reversed and then compute the integral :

$$\int_0^4 \int_0^{4-x} xy \, dy dx.$$

9. Calculate the Jacobian of transformation from rectangular to polar coordinates and hence evaluate the integral :

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{9 - x^2 - y^2}} dx dy.$$

10. Find the volume V of the solid bounded above by the cylinder $y^2 + z = 4$ and below by $x^2 + 3y^2 = z$.

11. Evaluate the integral below, where D is the region bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$:

$$\iiint_D z \, dx \, dy \, dz.$$

P.T.O.

12. Let D be the region in the xy-plane that is bounded by the co-ordinate axes and the line $x + y = 1$. Use the suitable change of variable to compute the integral :

$$\iint_D \left(\frac{x-y}{x+y}\right)^6 dy\, dx.$$

Section III

13. State Green's theorem for simply connected regions. Use Green's theorem to find the work done by the force field $\mathbf{F}(x, y) = (e^x - y^3)\boldsymbol{i} + (\cos y + x^3)\boldsymbol{j}$ along the circle $x^2 + y^2 = 1$ in anticlockwise direction.

14. Give the geometrical interpretation of the surface integral $\iint ds$ over piecewise smooth surface S. Evaluate the surface integral $\iint xz\, ds$ over the surface S which is the part of the plane $x + y + z = 1$ that lies in the first octant.

15. Verify Stokes' theorem for the vector field $\mathbf{F}(x, y, z) = 2z\boldsymbol{i} + 3x\boldsymbol{j} + 5y\boldsymbol{k}$ taking surface σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy-plane.

16. State and prove Divergence theorem.

17. Verify that the vector field $\mathbf{F}(x, y) = (e^x \sin y - y)\boldsymbol{i} + (e^x \cos y - x - 2)\boldsymbol{j}$ is conservative using cross partial test. Use a line integral to find the area enclosed by the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

18. Let E be the solid unit cube with opposing corners at the origin and (1, 1, 1) with faces parallel to co-ordinate planes. Let S be the boundary surface of E oriented with the outward pointing normal. If $\mathbf{F}(x, y, z) = 2xy\boldsymbol{i} + 3ye^{z}\boldsymbol{j} + x \sin z\,\boldsymbol{k}$, find the integral $\iint \mathbf{F}.\mathbf{n}\, ds$ over surface S using divergence theorem.

(6)

This question paper contains 2 printed pages.

Your Roll No. 5/12/17

Sl. No. of Ques. Paper: **5709** **H**

Unique Paper Code : **235301**

Name of Paper : **Calculus – II (MAHT-301)**

Name of Course : **B.Sc. (Hons.) Mathematics**

Semester : **III**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All Sections are compulsory.
Attempt any five questions from each Section.

SECTION I

1. Let f be the function defined by $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$ for $(x,y) \neq (0,0)$.
   a. Find $\lim_{(x,y)\to(2,1)} f(x,y)$.
   b. Prove that f has no limit at (0,0).
2. Compute the equation of the line tangent to graph of $f(x,y) = x^2 + xy + y^2$ in the x direction at the point (2, 3).
3. The radius and height of a right circular cone are measured with errors of atmost 3% and 2% respectively. Use increments to approximate the maximum possible percentage error in computing the volume of the cone using these measurements and the formula $V = \frac{1}{3}\pi R^2 H$.
4. If $f(x,y,z) = xyz + x^2y^3z^4$. Show that
$$f_{xyz} = f_{yzx} = f_{zxy}.$$
5. Find the tangent plane and normal line at $(1,-1,2)$ on the surface
$$S: x^2y + y^2z + z^2x = 5$$
6. Find the absolute extrema of $f(x,y) = e^{x^2-y^2}$ over $x^2 + y^2 \leq 1$.

SECTION II

7. Evaluate $\iint_T (x+2y+1)\,dA$, if T is the triangle in the x-y plane with vertices (0,0), (1,0) and (0,1).
8. Find the volume of the solid bounded above by the plane $z = y$ and below in the xy-plane by the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant.

P. T. O.

9. Find the volume of the solid formed by the portion of the paraboloid $z = 1 - x^2 - y^2$ that lies above the x- axis.

10. Compute the area in polar form of the region D bounded above by the line $y = x$ and below by the circle $x^2 + y^2 - 2y = 0$.

11. Using cylindrical coordinates find the volume of the solid in the first octant that is bounded by the cylinder $x^2 + y^2 = 2y$, the half-cone $z = \sqrt{x^2 + y^2}$, and the xy-plane.

12. Compute the volume of the solid Q that lies below the sphere $x^2 + y^2 + z^2 = 9$ and above the cone $z = \sqrt{x^2 + y^2}$.

SECTION III

13. Is the vector field $\vec{F}=(z^2 + 2xy - 1, x^2 + ze^y + 2, 2xz + e^y)$ the gradient of some function $f(x, y, z)$? If so find it?

14. Show that $f(x, y) = e^x \cos y$ is harmonic.

15. A wire has the shape of the curve

$$x = \sqrt{2} \sin t \qquad y = \cos t \qquad z = \cos t \qquad \text{for } 0 \le t \le \pi$$

If the wire has density $\delta(x, y, z) = xyz$ at each point(x, y, z), what is its mass?

16. Show that $\vec{F}=(3x^2yz + zye^{xz}, x^3z + e^{xz}, x^3y + xye^{xz})$ is a conservative vector field.

17. Find the flux of the vector field $F = zi + xj + (y + z)k$ through the parameterized surface

$$R(u, v) = (uv)i + (u - v)j + (2u + v)k$$

over the triangular region D in the uv-plane that is bounded by $u = 0, v = 0$, and $u + v = 1$.

18. Let S be the portion of the plane $x + y + z = 1$ that lies in the first octant, and let C be the boundary of S, traversed counterclockwise as viewed from above. Verify Stoke's theorem for the surface S and the vector field

$$F = -\frac{3}{2}y^2 i - 2xyj + yzk .$$

This question paper contains 3 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper: 5710 H

Unique Paper Code : 235302

Name of Paper : Numerical Methods and Programming (MAHT-302)

Name of Course : B.Sc. (Hons.) Mathematics

Semester : III

Duration : 3 hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory.
Attempt any two parts from each question.
Marks are indicated against each question.
Choice is given within the question.
Use of Scientific Calculator is allowed.

1. (a) Perform three iterations of Newton's method to find approximate value of $13^{1/3}$ using equation $x^3-13=0$ and starting approximation as 2.

(b) Verify that the function $x^5 + 2x - 1 = 0$ has a root in the interval (0, 1). Perform three iterations of bisection method.

(c) Explain order of convergence of an iterative method for finding an approximation to the location of a root of f(x)=0. Find order of convergence of the secant method.(13)

2. (a) Verify that the function $f(x) = x^3 + 2x^2 - 3x - 1$ has a zero on the interval (1, 2). Perform three iterations by using secant method to approximate the root.

(b) Perform three iterations starting from $p_0=1$ of the fixed point iteration scheme for $g(x) = e^{-x}$. Construct an algorithm to implement the fixed point iteration scheme.

(c) Write down an algorithmfor false position method. Perform three iterations of false-position method to approximate the root of the function $\cos(x) - xe^{x}=0$ in the interval (0, 1).

(13)

3. (a) Find an LU decomposition of the matrix

P. T. O.

$$A = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{bmatrix}$$

and use it to solve the system $Ax = [-4 \quad -10 \quad 9]^T$.

(b) Perform three iterations of Jacobi method to solve the system of equations, for the given coefficient matrix and right hand side vector, starting with the initial vector

$$x^0 = 0; \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}.$$

(c) Starting with initial vector $x^{(0)} = 0$, perform three iterations of Gauss Seidel method

to solve the following system of equations:

$2x - y = -1, -x + 4y + 2z = 3, 2y + 6z = 5.$ (13)

4 (a) Define the forward difference operator (Δ) and average operator (μ). Prove that Newton

Divided difference $f[x_0, x_1, x_2, \ldots\ldots x_n] = \frac{1}{n!h^n}\Delta^n f_0$.

(b) Find the backward difference polynomial that fits the data.

x	-1	2	4	5
$f(x)$	-5	13	255	625

Hence, interpolate at $x = 3.0$.

(c) Define the backward difference operator (∇) and central difference operator (δ).

Prove that:

(i) $E = 1 + \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$ (ii) $\mu = \sqrt{1 + \frac{\delta^2}{4}}$ (12)

5 (a) Apply Euler's method to approximate the solution of the initial value problem.

$\frac{dx}{dt} = tx^3 - x, \quad 0 \le t \le 1, \qquad x(0) = 1$

over the interval $[0,1]$ using four steps.

(b) Evaluate $\int_0^1 \tan^{-1} x\,dx$ using

(i) Simpson's one third Rule (ii) Trapezoidal Rule

(c) Find the Lagrange form of interpolating polynomial for the given data set.

x	-1	0	1	2
$f(x)$	4	1	2	3

Hence interpolate at $x = 1.5$.

(12)

6 (a) Verify that the forward difference approximation:

$$f'(x_0) = \frac{1}{2h}(-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h))$$

For the first order derivative provide the exact value of the derivative, regardless of the value of h, for the function $f(x) = 1$, $f(x) = x$, $f(x) = x^2$ but not for the function

$$f(x) = x^3.$$

(b) Define degree of precision of a quadrature rule. State Trapezoidal's rule for the evaluation of $\int_a^b f(x)dx$ and verify that it has degree of precision 1.

(c) If $f(x) = \frac{1}{x}$ then evaluate Newton Divided difference $f[a, b, c, d]$. **(12)**

This question paper contains 2 printed pages. 2017

Your Roll No. ....

H

Sl. No. of Ques. Paper: **5711**

Unique Paper Code : **235304**

Name of Paper : **Algebra – II (MAHT-303)**

Name of Course : **B.Sc. (Hons.) Mathematics**

Semester : **III**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do any two parts from each questions.

Questions

1. (a) Let $G = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in \mathbb{R}, a \neq 0 \right\}$. Show that G is a group under matrix multiplication. **(6)**

   (b) (i) Prove that if G is a group with the property that square of every element is identity then G is abelian.

   (ii) Define center of a group G. Show that center of a group G is an abelian subgroup of G. **(2 + 4)**

   (c) Define order of an element. Consider the element $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. What is the order of A in (i) $SL(2, \mathbb{R})$ (ii) $SL(2, \mathbb{Z}_p)$, p is a prime. **(6)**

2. (a) Let $G = \langle a \rangle$ be a cyclic group of order n. Prove that $G = \langle a^k \rangle$ if and only if $\gcd(n, k) = 1$. Find all the generators of Z_{20}. **(6.5)**

   (b) Suppose that a and b are group elements that commute have orders m and n respectively. If $\langle a \rangle \cap \langle b \rangle = \{e\}$. Prove that the group contains an element whose order is the least common multiple of m and n. Show that this need not be true if a and b do not commute. **(6.5)**

(c) Let 'a' be a fixed element of a group G. Define centralizer of the element a. Show that $Z(G) = \bigcap_{a \in G} C(a)$. (6.5)

3. (a) (i) Prove that product of two odd permutation is an even permutation.

(ii) Show that $Z(S_n) = \{\varepsilon\}$ for $n \geq 3$. (2 + 4)

(b) Show that if H is a subgroup of S_n then every member of H is an even permutation or exactly half of them are even. (6)

(c) (i) Let H and K be subgroups of a group G. If $|H| = 12$ and $|K| = 35$, find $|H \cap K|$.

(ii) Find all left cosets of {1, 11} in U(30). (2 + 4)

4. (a) State and prove Lagrange's theorem for finite groups. (6.5)

(b) (i) Prove that every subgroup of D_n of odd order is cyclic.

(ii) Prove or disprove Z x Z is a cyclic group. (3.5 + 3)

(c) Define index of a subgroup in a group. Show that Q, the group of rational numbers under addition has no proper subgroup of finite index. (6.5)

5. (a) Let G be a group and H a normal subgroup of G. The set $G/H = \{aH | a \in G\}$ is a group under the operation (aH) (bH) = abH. (6)

(b) Let N be a normal subgroup of a finite group G. If N is cyclic, prove that every subgroup of N is normal in G. (6)

(c) Determine all the homomorphisms from Z_{12} to Z_{30}. (6)

6. (a) Suppose that φ is an isomorphism from a group G onto a group G^*. Prove that G is cyclic if and only if G^* is cyclic. Hence show that Z, the group of integers under addition is not isomorphic to Q, the group of rationals under addition. (6.5)

(b) State and prove Cayley's theorem. (6.5)

(c) Let M and N be normal subgroups of a group G and $N \subseteq M$. Prove that $(G/N)/(M/N) \cong G/M$. (6.5)

This question paper contains ? printed pages

Unique paper code : 2351301

Name of the course : B. Sc. (Hons) Mathematics

Name of the paper : Algebra II (Group Theory - I)

Semester : III

Duration : 3 Hours

Maximum marks : 75

Roll No.

4/12/17

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of the question paper.
2. Attempt **any two parts** from each question.
3. All questions are compulsory.

1(a) (i) For any elements a and b from a group and any integer n prove that $(a^{-1}ba)^n = a^{-1}b^n a$.

(ii) Give an example of a non-cyclic group all of whose proper subgroups are cyclic.

(b) Define center of a group. Prove that the center of a group G is a subgroup of G.

(c) If $G = \langle a \rangle$ is a cyclic group of order n then prove that $G = \langle a^k \rangle$ *iff* g.c.d $(k, n) = 1$.

(6 × 2 = 12)

2(a) Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in \mathbb{R}, a \neq 0 \right\}$. Show that G is a group under matrix multiplication.

(b) (i) Let H be a non empty finite subset of a group G. Then prove that H is a subgroup of G if H is closed under the operation of G.

(ii) Let G be a group and let a be any element of G. Then prove that $\langle a \rangle$ is subgroup of G.

(c) (i) How many subgroup does Z_{30} have. List a generator for each of these.

(ii) Prove that $H = \left\{ \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \mid n \in \mathbb{Z} \right\}$ is a cyclic subgroup of $GL(2, \mathbb{R})$ (6 × 2 = 12)

3(a) Prove that the order of a permutation of a finite set written as a product of disjoint cycles, is the least common multiple of the lengths of the cycles.

(b) State and prove Lagrange's Theorem. Is the converse true? Justify your answer.

(c) Let G be a group and H a normal subgroup of G. Prove that the set $G/_H = \{aH | a \in G\}$ is a group under the operation $(aH)(bH) = abH$.

(6 × 2 = 12)

4. (a) Show that if H is a subgroup of S_n (n ≥ 2) then either every member of H is an even permutation or exactly half of them are even.

(b) State and prove Fermat's Little Theorem.

(c)(i) Prove that a subgroup H of a group G is a normal subgroup of G if and only if

$ghg^{-1} \in H$ for all $g \in G$ and for all $h \in H$.

(ii) Suppose G is a group and H = $\{g^2 : g \in G\}$ is a subgroup of G. Prove that H is a normal subgroup of G.

(6.5 × 2 = 13)

5. (a) Let G be a group and Z (G) be the centre of G. If G/Z (G) is cyclic then prove that G is Abelian.

(b) Show that any infinite cyclic group is isomorphic to (Z, +) the group of integers under addition.

(c) Let G be a group of permutation and {1,-1} be the multiplicative group. For each $\sigma \in G$, define a mapping

$\varphi : G \rightarrow \{1, -1\}$,

by

$$\varphi(\sigma) = \begin{cases} 1 & \text{if } \sigma \text{ is an even;} \\ -1 & \text{if } \sigma \text{ is an odd.} \end{cases}$$

Prove that φ is a group homomorphism. Also, find Ker φ.

(6.5 × 2 = 13)

6. (a) Suppose that φ is an isomorphism from a group G onto a group G*. Prove that G is cyclic if and only if G* is cyclic. Hence show that Z, the group of integers under addition is not isomorphic to Q, the group of rationals under addition.

(b) If M and N are normal subgroups of a group G and N ≤ M, prove that (G/N) / (M/N) ≈ G/M.

(c) Let φ be a group homomorphism from G onto G* then prove that G/Ker φ ≈ G*.

(6.5 × 2 = 13)

(10)

Sl. No. of Q.P. : 5986 4|12|17

Unique Paper Code : 2351302

Name of the Paper : Analysis-II (Real Functions)

Name of the Course : Maths(H) – II

Semester : III

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

*Attempt any **three** parts from each question.*

1. a. Let $A \subseteq R$ and $c \in R$ be a cluster point of A and $f: A \to R$, then define limit of function f at c. Also show that if f has a limit at $c \in R$, then f is bounded on some neighbourhood of c. 5

b. Let $c \in R$. Use ε-δ definition to show that $\lim_{x \to c} x^3 = c^3$. 5

c. State Sequential Criterion of Limits. Using sequential criterion, prove that $\lim_{x\to 0} \frac{1}{x^2}$, $x > 0$ does not exist. 5

d. Let $A \subseteq R$, let $f, g, h: A \to R$ and let $c \in R$ be a cluster point of A. If $f(x) \leq g(x) \leq h(x)$ for all $x \in A, x \neq c$, and if $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$, then show that $\lim_{x\to c} g(x) = L$. 5

2. a. Let $A \subseteq R$, let f and g be functions from A to R and let $c \in R$ be a cluster point of A. Suppose that f is bounded in a neighbourhood of c and $\lim_{x\to c} g(x) = 0$. Prove that $\lim_{x\to c}(fg)(x) = 0$. 5

b. Let $f(x) = \frac{1}{(e^{1/x} + 1)}$ $for\ x \neq 0$, then find $\lim_{x\to 0-} f(x)$ and $\lim_{x\to 0+} f(x)$. 5

c. Determine the points of continuity of the function $f(x) = [\![x]\!], x \in R$, where $[\![x]\!]$ denotes the greatest integer $n \in \mathbb{Z}$ such that $n \leq x$. 5

d. Let $g: R \to R$ be defined by

$$g(x) = \begin{cases} x, & \text{for } x \text{ rational} \\ 0, & \text{for } x \text{ irrational} \end{cases}$$

Find all the points at which g is continuous. **5**

3. a. Let $A \subseteq R$ and $f: A \to R$ such that $f(x) \geq 0$ for all $x \in A$. Show that if f is continuous at $c \in A$, then $\sqrt{f}$ is continuous at c. **5**

b. Let $A \subseteq R$. Let $f: A \to R$ and $g: A \to R$ be continuous on A. Show that $f + g$ is continuous on A. **5**

c. Let $f: R \to R$ be continuous on R and let $P = \{x \in R: f(x) > 0\}$. If $c \in P$ show that there exists a neighbourhood $V_\delta(c) \subseteq P$. **5**

d. Suppose that f is a real valued continuous function on R and that $f(a)f(b) < 0$ for some $a, b \in R$. Prove that there exists x between a and b such that $f(x) = 0$. Prove that $x = \cos x$ for some x in $(0, \frac{\pi}{2})$. **5**

4. a. Show that every uniformly continuous function on $A \subseteq R$ is continuous on A. Is the converse true? Justify your answer. **5**

sin x

b. Show that the function $\sin(1/x), x \neq 0$ is ~~not~~ uniformly continuous on $(0, \infty)$. **5**

c. Let $f: R \to R$ be defined by

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that f is differentiable at $x = 0$ and find $f'(0)$. **5**

d. Let $f: I \to R$ is differentiable on the interval I. Prove that f is increasing on I if and only if $f'(x) \geq 0$ for all $x \in I$. **5**

5. a. Use the Mean Value theorem to prove $(x-1)/x < \ln x < x - 1$ for $x > 1$. **5**

b. For the function $f: R \to R$ given by $f(x) = 3x - 4x^2$, find the points of relative extrema. Also find the intervals on which the function is increasing and those on which it is decreasing. **5**

c. State and prove Cauchy's Mean Value theorem. **5**

d. Obtain Maclaurin's series expansion for the function $f(x) = \cos x, x \in R$ **5**

(11)

5/12/17

Sl. No: ... 6012

Set-A

Unique Paper Code: 2352301

Name of the Paper: Calculus

Name of the Course: Allied Course

Semester: 3

Duration: 3 hours

Maximum Marks: 75 Marks

Instructions for Candidates

- Attempt five questions from each section.
- Each question carries five marks.

Section: 1

1. Use ε-δ definition to show $\lim_{x\to 1}(5x-3)=2$.
2. Find the horizontal and vertical asymptotes of the curve $y=\frac{x+3}{x+2}$.
3. Sketch the graph of $f(x)=\frac{(x+1)^2}{1+x^2}$.
4. Evaluate $\lim_{x\to\infty}\frac{\ln x}{2\sqrt{x}}$.
5. Find the volume of the solid generated by revolving the region between the y-axis and the curve $x=\frac{2}{y}$, $1\le y\le 4$, about the y-axis.
6. Find the length of the curve $y=\frac{4\sqrt{2}}{3}x^{3/2}-1$, $0\le x\le 1$.

Section-2

7. Find a polar equation for the curve $x^2+(y-3)^2=9$.
8. Graph the curve $r^2=4\cos\theta$.
9. A glider is soaring upward along the helix $r(t)=(\cos t)\,i+(\sin t)\,j+t\,k$. How long is the glider's path from $t=0$ to $t=2\pi$?
10. Find the curvature of the curve $r(t)=(a\cos t)i+(a\sin t)j$.
11. Find the principal unit normal vector for the motion $r(t)=(\cos 2t)i+(\sin 2t)j$.
12. Evaluate $\lim_{(x,y)\to(0,0)}\frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$.

Section-3

13. Show that $f(x,y)=\frac{2x^2y}{x^4+y^2}$ has no limit as (x,y) approaches $(0,0)$. does not exist.
14. Find f_{yxyz} if $f(x,y,z)=1-2xy^2z+x^2y$.
15. Find $\frac{dy}{dx}$ if $y^2-x^2-\sin xy=0$.

lim (x,y)→(0,0)

(1)

16. Find the derivatives of $f(x, y) = x^2 + xy$ at point (1, 2) in the direction of the vector $u = \left(\frac{1}{\sqrt{2}}\right) i + \left(\frac{1}{\sqrt{2}}\right) j$.
17. Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9$ at the point (1, 2, 4).
18. Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$